THE SPACE-TIME ASYMMETRY RESEARCH (STAR) PROGRAM

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ABSTRACT

Space-Time Asymmetry Research (STAR) is a proposed satellite mission that aims for significantly improved tests of fundamental space-time symmetry and the foundations of special and general relativity. In the current concept, STAR comprises a series of three subsequent missions with increasingly advanced instruments performing clock to clock comparisons. While the first STAR missions will perform Kennedy-Thorndike (KT) and Michelson-Morley (MM) experiments, later missions will focus on fundamental gravitational physics by precision measurement of gravitational redshift, time dilation and Local Position Invariance (LPI). Compared to previous experimental accuracy, STAR aims for an improvement of at least two orders of magnitude. The STAR1 mission will measure the constancy of the speed of light to one part in 10^{-17} and derive the Kennedy Thorndike coefficient of the Mansouri-Sexl test theory to 7×10^{-10}. The KT experiment will be performed by comparison of an atomic or molecular frequency reference with a length reference (highly stable cavity made e.g. from ultra low expansion (ULE) glass ceramics) during flight around Earth with an orbital velocity of 7 km/s. The corresponding sensitivity to a boost dependent violation of Lorentz invariance as modeled by the KT term in the Mansouri-Sexl test theory or a Lorentz violating extension of the standard model (SME) will be significantly enhanced as compared to Earth-based experiments. The space environment will enhance the measurement precision such that an overall improvement by a factor of 400 over current Earth bound experiments is expected. The STAR1 philosophy is to realize a fast, small – and therefore cheap – mission with a high scientific output, also providing the instrument technology and the spacecraft for the subsequent STAR missions, which plan to use different optical frequency standards. The 180 kg small satellite will be attitude, vibration and temperature controlled. The power consumption of the whole spacecraft will be less than 185 W. The launch of STAR1 is foreseen for 2015, the follow-on missions will be flown with an overlap with the previous mission by two to three years. Each mission has a maximum duration of 5 years (from mission set up to data acquisition) which permits students to experience the full mission lifecycle. Education and training of undergraduate and graduate students is a specific mission goal.

INTRODUCTION

The theories of special relativity (SR) and general relativity (GR) are the basis of our physical understanding of space and time. Modern theories of quantum gravity predict very small deviations from special and general relativity, motivating high-accuracy tests of SR and GR and their foundations. Earth-based tests, including

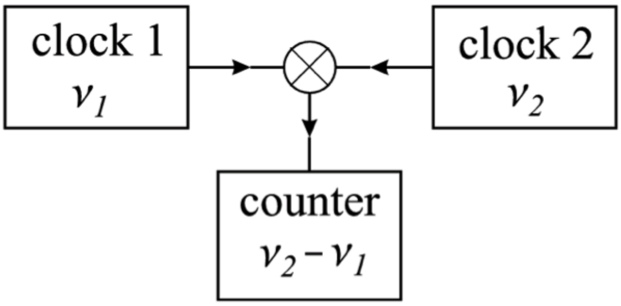

Figure 1. Tests of relativity based on clock-to-clock comparison experiments. The beat signal is analyzed when changing parameters as the velocity v, the orientation θ or the gravitational field g of the setup (see Theoretical Description). The clocks can either be a length reference (cavity) or an atomic (or molecular) reference.

Michelson-Morley and Kennedy-Thorndike experiments as well as tests of general relativity (such as universality of gravitational redshift, time dilation) are carried out in numerous laboratories. Significant improvement in accuracy can only be realized by satellite-based experiments. An optical test of SR and GR was already proposed in 2001 as the OPTIS mission [1, 2, 3], a first feasibility study was carried out on behalf of the DLR.

Here, we present a new mission concept which is partly based on the OPTIS expertise. The Space-Time Asymmetry Research (STAR) program will carry out tests of special and general relativity using advanced optical metrology technologies. The international collaboration for the STAR mission currently includes universities and institutions in the USA, Saudi-Arabia, Great Britain and Germany. Beside tests of fundamental physics, STAR also aims at developing small satellite and advanced instrumentation technology as well as educating future scientists and engineers.

OVERVIEW OVER STAR

The STAR mission program consists of a series of three subsequent missions with progressively advanced payload instrumentation using the same spacecraft and bus system. The mission lifetime is supposed to be >1 year. Clock-clock-comparison experiments (compare Fig. 1) will be carried out for tests of special and general relativity with up to a factor of 1000 improved accuracy compared to ground-based experiments. The satellite payload will include lasers frequency stabilized to high-finesse cavities and atomic (or molecular) optical references. The first mission will utilize atomic references based on Doppler-free spectroscopy of I_2 (e. g. using modulation-transfer spectroscopy (MTS), frequency-modulation spectroscopy (FMS), fluorescent spectroscopy, polarization spectroscopy) and/or noise-immune cavity-enhanced optical heterodyne molecular spectroscopy (NICE-OHMS) of CO or C_2HD. Thermal atomic beam, single-ion and lattice clocks – in combination with an optical frequency comb – are proposed for the later missions for enhanced accuracy in the science measurements. Comparison between two cavity stabilized lasers will result in a Michelson-Morley experiment, while comparison between a cavity stabilized laser and an atomic (or molecular) standard will yield a Kennedy-Thorndike experiment. STAR1 will be flown in a circular low-Earth orbit (LEO), follow-on missions in an elliptical orbit where the change in gravitational field over the orbit will be used for gravitational redshift experiments.

The STAR project is an international cooperative effort of teams with very strong background in experimental tests of relativity, their theoretical description and advanced space technology. The mission lead is carried out by Stanford University (USA), the cooperation further includes NASA Ames Research Center (USA), King Abdulaziz City of Science and Technology (KACST, Saudi-Arabia), Birmingham University (UK) – and a German team consisting of the DLR-Institute for Space Systems, the Center for Applied Space Technology and Microgravity ZARM (Bremen), the Humboldt-University Berlin and the University of Applied Sciences (HTWG) Konstanz.

THEORETICAL DESCRIPTION

To test space and time, length and time references are necessary. Modern experiments testing special and general relativity use cavities whose resonance frequency is given by $\nu_{res} = nc/2L$ where L is the cavity length and n an integer number. Following the test theory of H.P. Robertson [4], later extended by R. Mansouri and

R.U. Sexl [5], a velocity and orientation dependency of the speed of light $c = c(v, \theta)$, and thus of the cavity resonance frequency, can be modeled as:

$$\frac{c(v,\theta)}{c} = 1 + \underbrace{(\beta - \alpha - 1)}_{A} \frac{v^2}{c^2} + \underbrace{\left(\frac{1}{2} - \beta + \delta\right)}_{B} \frac{v^2}{c^2} \sin^2\theta \,. \tag{1}$$

The Robertson-Mansouri-Sexl (RMS) framework assumes the existence of a preferred frame, usually taken to be the Cosmic Microwave Background (CMB) and the velocity v is taken with respect to this frame. θ corresponds to the angle between the velocity vector and the direction of light propagation i.e. the cavity axis. The first (velocity dependent) term, with coefficient A, can be tested in a Kennedy-Thorndike (KT) experiment, the second (orientation dependent) term, with coefficient B, in a Michelson-Morley (MM) experiment. In special relativity $\beta = 1/2$, $\alpha = -1/2$ and $\delta = 0$, and therefore A and B vanish. Apart from the RMS formalism, boost and orientation dependency of c can be modelled within other test theories as well. Most importantly, a Lorentz violating extension of the Standard Model developed by V. A. Kostelecky and coworkers [6] has been used for the analysis of previous MM and KT experiments (see [7] and [8] for an overview).

Tests of Special Relativity

For testing a dependency of the speed of light on the velocity of the laboratory, a cavity stabilized laser can be used. Its resonance frequency depends on the length L_1 of the cavity. The velocity of the cavity is changed over time and its resonance frequency is analyzed when compared to an atomic reference laser or a second laser stabilized to a cavity with length $L_2 \neq L_1$. Earth-based experiments rely on the rotation of the Earth around its axis or the revolution of the Earth around the Sun to vary the laboratory velocity. The corresponding velocities are $v = v_0 \pm 300\,\mathrm{m/s}$ for the Earth's rotation (taken for a laboratory at a geographical latitude of 50°) and $v = v_0 \pm 30\,\mathrm{km/s}$ for Earth's revolution around the Sun. The velocity $v_0 = 377\,\mathrm{km/s}$ is the velocity of the solar system with respect to the cosmic microwave background.

The currently most accurate result for the KT-parameter A has been obtained from the comparison of a (microwave) cryogenic sapphire oscillator with a H-maser over a period of over 6 years [9]. The data was analyzed with respect to sidereal, diurnal and annual modulation, resulting in the following upper limits:

$$|\beta - \alpha - 1| \leq 4 \cdot 10^{-8}, \qquad \frac{\delta_v c}{c} \leq 1 \cdot 10^{-16} \,. \tag{2}$$

The most accurate Kennedy-Thorndike experiment in the optical frequency domain has been carried out by comparing a cryogenic optical resonator to a laser, frequency stabilized to a hyperfine transition in molecular iodine [10]. Data was taken over a period of 190 days and analyzed with respect to the annual modulation.

Modern ground-based experiments of the isotropy of space typically use two crossed optical cavities mounted on a turntable. The beat signal between two lasers stabilized to the two cavities is analyzed at twice the rotation frequency. The currently most accurate results are published in [11] and [12], giving the following upper limits:

$$\left|\frac{1}{2} - \beta + \delta\right| \leq 8 \cdot 10^{-12}, \qquad \frac{\delta_\theta c}{c} \leq 1 \cdot 10^{-17} \,. \tag{3}$$

PRESENT STATUS

An overview over the progress in Kennedy-Thorndike and Michelson-Morley experiments is given in Fig. 2 and Fig. 3, respectively. The demonstrated accuracy is shown in δ_v/c for KT and δ_θ/c for MM experiment.

Advantages of Space

Performing satellite experiments offers several advantages: (i) large changes in velocity can be achieved: 7.4 km/s in LEO compared to 330 m/s on Earth, i.e. a factor of about 20 – directly improving the KT accuracy; (ii) the orbit time of 90 min is significantly shorter than the 24 h period on Earth, relaxing the frequency stability requirements of the clocks; (iii) high changes in gravitational potential can be realized in space using elliptical orbit, improving the accuracy of the test of the gravitational redshift; (iv) space environment reduces vibrations and minimizes distortions of the cavities; and (v) long integration times are possible (1 – 2 years, given by the mission lifetime).

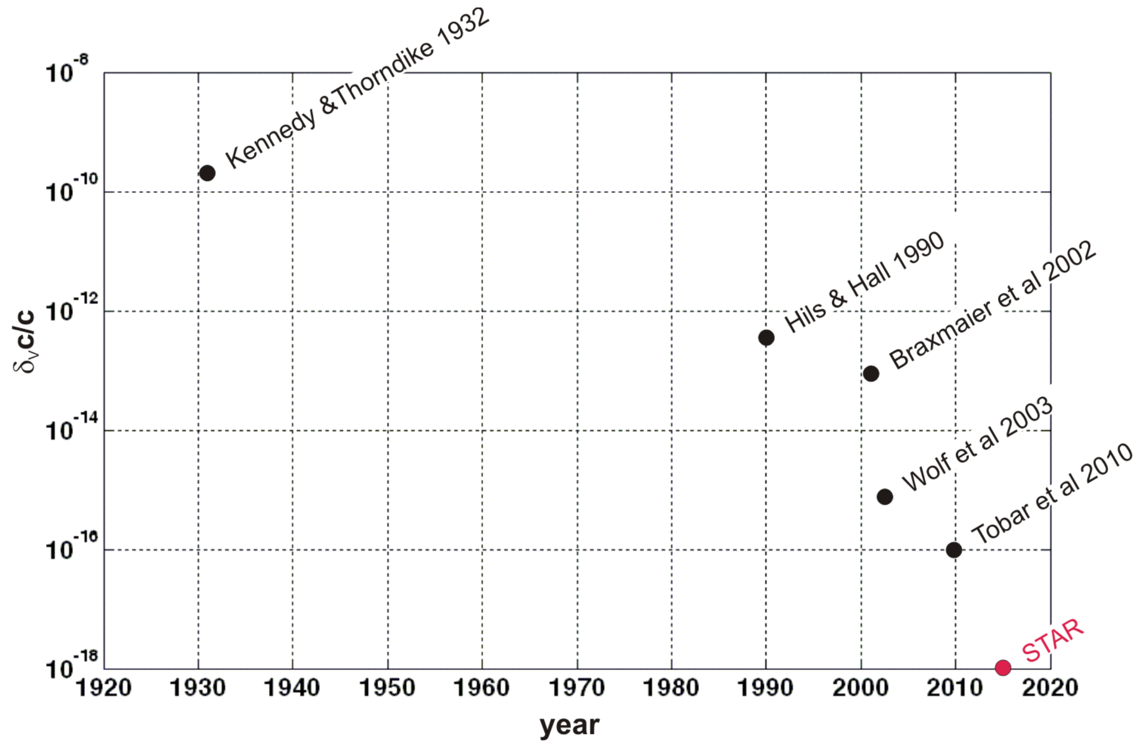

Figure 2. Progress of the Kennedy-Thorndike experiments.

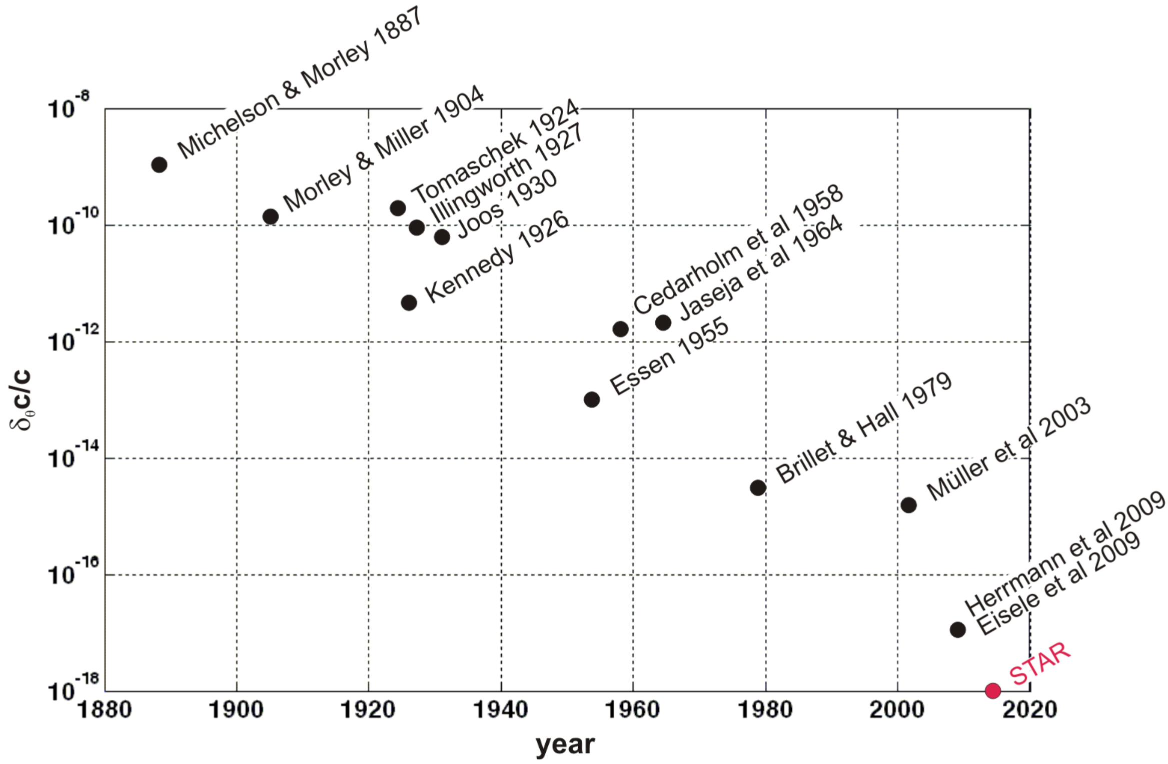

Figure 3. Progress of the Michelson-Morley experiments.

Table 1. Overview over the three STAR missions.

	STAR1	**STAR2**	**STAR3**
launch year	2015	2018	2021
science	KT, (MM)	KT, MM, GR	KT, MM, GR, others
KT accuracy	10^{-17}	10^{-18}	10^{-19}
MM accuracy	(10^{-17})	10^{-18}	10^{-19}
GR accuracy	–	multi-spectrum redshift	multi-spectrum multi-location redshift
instruments	clocks + cavities	clocks + cavities + comb	clocks + cavities + comb + link
clock type	MTS, NICE-OHMS	single-ion clock	lattice clock
wavelength	1064 nm or 1565 nm	multiple	multiple
cavity finesse	100.000	200.000	5.000.000
payload mass	80 kg	80 kg	80 kg
total spacecraft mass	180 kg	(180 kg)	(180 kg)
total spacecraft power	185 W	185 W	185 W
orbit	circular, LEO	elliptical	elliptical

THE STAR MISSION SERIES CONCEPT

The STAR mission program addresses three major goals:

1. perform tests of fundamental physics,
2. develop small satellite and advanced instrumentation technology,
3. educate future scientists and engineers.

STAR consists of three subsequent missions which will utilize advanced optical clock technology and perform clock-clock-comparison between a length and an atomic (or molecular) reference. The length reference is based on a laser stabilized to a cavity made of a material with very low coefficient of thermal expansion (CTE) such as Zerodur, ULE or Si. The atomic (or molecular) reference is based on an atomic, ionic or molecular transition with narrow linewidth and small systematic effects to reduce complexity.

A summary of the three missions is given in Table 1. A 180 kg small satellite is used as foundation for all three STAR missions. STAR1 is planned to be launched 2015, the follow-on missions in a 3 years interval. STAR2 and STAR3 have an overlap of 2 years with its respective precursor mission, guaranteeing personal and technological constancy. The time frame of one mission (approx. 5 years) corresponds to a typical Ph.D. time scale, enabling students to participate in the mission from mission design to integration, launch and data analysis. In order to realize the 2015 launch date for STAR1, the satellite will be based on reduced complexity, one clear science goal and the use of (laboratory demonstrated) state-of-the-art technology. STAR2 and STAR3 will use more advanced technologies whose development will be carried out parallel to STAR1.

The STAR1 mission will focus on a Kennedy-Thorndike experiment by comparing a cavity stabilized laser to an atomic reference. For redundancy purposes, both clocks will be realized twice. Therefore, in principle, a Michelson-Morley experiment can be carried out by comparing the two cavity stabilized lasers. The main scientific output of STAR1 is an improved Kennedy-Thorndike experiment, the add-on Michelson-Morley experiment is meant not to drive the mission requirements. A more detailed overview over the STAR1 payload is given in the following subsection.

STAR2 and STAR3 will perform (further) improved Kennedy-Thorndike and Michelson Morley experiments as well as tests of the general theory of relativity. The gravitational field the spacecraft is exposed to is varied in

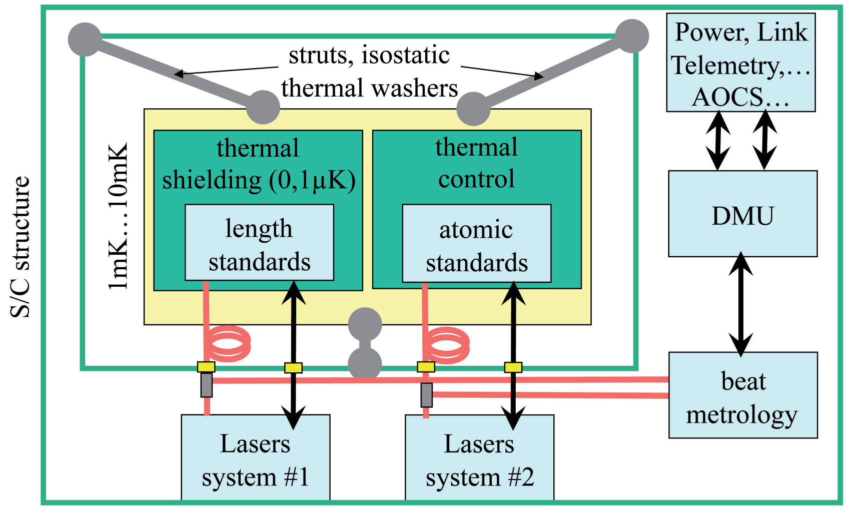

Figure 4. Functional diagram of the STAR-1 payload (DMU: data management unit). The required mK temperature stability of the optics payload can benefit from LISA / LISA Pathfinder heritage. Redundancy is not shown in the diagram.

an elliptical orbit. By comparing clocks of different type (e. g. cavity vs. atomic clock or two atomic clocks of different type), a test of the gravitational redshift and its universality can be carried out. An improved clock stability (compared to STAR1) is achieved using cavities with higher finesse and advanced materials (e. g. single crystalline silicon at low temperatures) and single-ion clocks or lattice clocks. An optical frequency comb will be used for clock comparison.

STAR1 Payload Concept

The functional diagram of the STAR1 payload is shown in Fig. 4. The length standard is realized by a laser stabilized to a high-finesse cavity probably made of ULE, a glass ceramic with a CTE of $3 \cdot 10^{-8}\,\mathrm{K}^{-1}$. Several possible realizations of the atomic (molecular) reference are currently investigated in more detail, based on two different spectroscopy methods: (i) modulation-transfer spectroscopy (MTS; or frequency modulation spectroscopy, FMS) of molecular iodine at a wavelength of 532 nm or 508 nm, and (ii) noise-immune cavity-enhanced optical heterodyne molecular spectroscopy (NICE-OHMS) of CO at a wavelength of 1565 nm or of C_2HD at a wavelength of 1064 nm. A comparison of the characteristics of the main spectroscopy methods is given in Table 2.

In a first step, a compact iodine stabilization setup using an NPRO-design Nd:YAG laser at 1064 nm will be realized on elegant breadboard level (EBB). Such a laser type is already available space-qualified by Tesat GmbH (Backnang, Germany) providing up to 1 W output power in the infrared. The frequency doubling will be realized using a periodically-poled KTP crystal, where radiation tests were already carried out [13]. The optical spectroscopy setup is proposed to use a Zerodur baseplate where the optical component are fixed to using bonding technology. This setup ensures high mechanical and thermal stability which is needed to withstand launch and shock and vibration tests. It also guarantees high pointing stability of the two counter-propagating laser beams in the gas cell which otherwise might limit the achieved long-term frequency stability. A long interaction pathlength in the gas cell is needed for high signal-to-noise ratio. In a laboratory setup at the Humboldt-University Berlin, a gas cell with a length of 0.8 m is used, which is passed thrice by pump and probe beams. A frequency stability below $1 \cdot 10^{-14}$ at integration times of 300 s is demonstrated [14]. For the compact Zerodur setup, the implementation of a multi-pass cell with maximum dimension of $10 \times 10 \times 10\,\mathrm{cm}^3$ is proposed.

In parallel, the setup of a NICE-OHMS standard will be developed. The use of CO is promising since a better frequency stability of 2 to $4 \cdot 10^{-16}$ is expected, in comparison to an iodine stabilized setup. On the other hand, CO at 1565 nm requires a space qualified laser diode which is currently not available commercial-of-the-shelf. Also, the very weak transitions necessitate the use of NICE-OHMS spectroscopy to saturate the transition which implies higher complexity compared to MTS. This includes the need of a cavity with related Pound-Drever-Hall locking technique. A fall-back option is the use of NICE-OHMS of C_2HD at a wavelength of 1064 nm where space-qualified Nd:YAG lasers can be used and a frequency stability of $1 \cdot 10^{-14}$ at an integration time of 600 s was already demonstrated in laboratory experiment [15].

Table 2. Comparison of spectroscopy methods for STAR1.

	I_2		CO	C_2HD
absorption wavelength	532 nm	508 nm	1565 nm	1064 nm
laser source	Nd:YAG (or laser diode)		laser diode	Nd:YAG (or laser diode)
second harmonic generation	yes		no	
spectroscopy	MTS, FMS, or fluorescent spectroscopy		NICE-OHMS	
gas temperature	(approx. $-10°$C)		room temperature	
clock stability	$4 \cdot 10^{-15}$ at 3000 s (lab demonstrated [16])	$1 \cdot 10^{-14}$ (estimated)	$2...4 \cdot 10^{-16}$ (estimated)	$1 \cdot 10^{-14}$ at 600 s (lab demonstrated [15])

The cavity setup will include two cavities for redundancy which are proposed to be realized as two crossed cavities in one monolithic ULE block. The main criticality is the thermal requirement where a first estimate yields to a required $0.1\,\mu$K to $1\,\mu$K temperature stability for obtaining a frequency stability of $1 \cdot 10^{-15}$. As the absolute temperature is not critical, a passive shielding using multi-layer insulation foil is assumed to be adequate.

STAR2 and STAR3: Optical Clock and Frequency Comb Development

The follow-on missions STAR2 an STAR3 will use atomic references with improved frequency stability below 10^{-15} in combination with an optical frequency comb. STAR3 also includes a downlink for comparison with Earth-based clocks. Candidate clocks are thermal atomic beam clocks, single-ion and lattice clocks. They are all state-of-the-art laboratory technologies which demonstrated the required frequency stability.

Clocks based on thermal atomic beams represent a possible atomic standard for STAR2 which can be realized very compact. They state a well-established technology e.g. based on laser-cooled neutral Ca atoms [17]. STAR3 will use single-ion or lattice clocks. Single-ion clocks using Hg^+, Al^+, Yb^+ and Sr^+ ions are demonstrated in laboratory setups. Lattice clocks using neutral Sr and Yb atoms are also possible clocks for STAR3.

Optical frequency combs can guarantee optical frequency comparisons to a few parts in 10^{-19}. A frequency comb for drop-tower application with peak decelerations up to 50 g has been developed by Menlo Systems GmbH (Martinsried, Germany) and is tested by the Center for Applied Space Technology and Microgravitation (ZARM, Bremen) in a DLR project. Femto-second laser technology for space applications is currently developed in a cooperation of the University of Applied Sciences Konstanz (HTWG), the University Konstanz and Astrium GmbH – Satellites (Friedrichshafen).

CONCLUSION AND SUMMARY

We presented the STAR mission program which consists of three subsequent missions performing space-based tests of special and general relativity with an up to a factor of 1000 improved accuracy over ground-based experiments. In space, they benefit from quiet environment, long integration times, large velocities and large changes in gravitational potential. The STAR missions will use small satellite and advanced instrumentation technology, one specific mission goal is the education of future scientists and engineers. The payload will use optical clocks – i. e. cavity stabilized lasers, references based on Doppler-free absorption spectroscopy, thermal beams, cooled atoms, trapped ions or lattice clocks – in combination with an optical frequency comb.

ACKNOWLEDGMENTS

The STAR mission is a collaboration of institutions in the USA, Saudi-Arabia, Great Britain and Germany. The friendly, motivating and effective atmosphere is highly appreciated. The authors thank Robert Byer, Sasha Buchman, John Lipa and Ke-Xun Sun from Stanford University, John Hall from NIST, Peter Worden, Belgacem Jaroux and Brian Lewis from NASA Ames and Turki Al-Saud and Haithem Altwaijry from KACST for their support.

REFERENCES

[1] C. Lämmerzahl, H. Dittus, A. Peters, and S. Schiller. OPTIS: a satellite-based test of special and general relativity. *Class. Quantum Grav.*, 18:2499–2508, 2001.

[2] C. Lämmerzahl, I. Ciufolini, H. Dittus, L. Iorio, H. Müller, A. Peters, E. Samain, S. Scheithauer, and S. Schiller. OPTIS – An Einstein mission for improved tests of Special and General Relativity. *Gen. Rel. Grav.*, 36:2373, 2004.

[3] S. Scheithauer, C. Lämmerzahl, H. Dittus, S. Schiller, and A. Peters. The OPTIS satellite – improved tests of Special and General Relativity. *Aerospace Science and Technology*, 9:357–365, 2005.

[4] H. P. Robertson. Postulate versus Observation in the Special Theory of Relativity. *Rev. Mod. Phys.*, 21(3):378–382, 1949.

[5] R. Mansouri and R. U. Sexl. A test theory of special relativity. *Gen. Rel. Grav.*, 8:495, 515, 809, 1976.

[6] D. Colladay and V. A. Kostelecký. Lorentz-violating extension of the standard model. *Phys. Rev. D*, 58:116002, 1998.

[7] D. Mattingly. Modern tests of lorentz invariance. *Living Reviews in Relativity*, 8(5), 2005.

[8] G. Amelino-Camelia, C. Lämmerzahl, A. Macias, and H. Müller. The search for quantum gravity signals. In A. Macias, C. Lämmerzahl, and D. Nunez, editors, *Gravitation and Cosmology*, page 30. AIP Conference Proceedings 758, Melville, New York, 2005.

[9] M. E. Tobar, P. Wolf, S. Bize, G. Santarelli, and V. Flambaum. Testing local Lorentz and position invariance and variation of fundamental constants by searching the derivative of the comparison frequency between a cryogenic sapphire oscillator and hydrogen maser. *Phys. Rev. D*, 81(2):022003, 2010.

[10] C. Braxmaier, H. Müller, O. Pradl, J. Mlynek, and A. Peters. Test of Relativity Using a Cryogenic Optical Resonator. *Phys. Rev. Lett.*, 88:010401, 2002.

[11] S. Herrmann, A. Senger, K. Möhle, M. Nagel, E. V. Kovalchuk, and A. Peters. Rotating optical cavity experiment testing Lorentz invariance at the 10^{-17} level. *Phys. Rev. D*, 80:105011, 2009.

[12] C. Eisele, A. Y. Nevsky, and S. Schiller. Laboratory Test of the Isotropy of Light Propagation at the 10^{-17} Level. *Phys. Rev. Lett.*, 103:090401, 2009.

[13] V. Leonhardt and J. B. Camp. Space interferometry application of laser frequency stabilization with molecular iodine. *Appl. Optics*, 45(17):4142–4146, 2006.

[14] K. Döringshoff, K. Möhle, M. Nagel, E. V. Kovalchuk, and A. Peters. High performance iodine frequency reference for tests of the LISA laser system. *see these Proceedings.*

[15] J. Ye, L.-S. Ma, and J. L. Hall. Ultrastable Optical Frequency Reference at 1.064 μm Using a C_2HD Molecular Overtone Transition. *IEEE Trans. Instrum. Meas.*, 46(2):178–182, 1997.

[16] E. J. Zang, J. P. Ciao, Y. Li, C. Y. Li, Y. K. Deng, and C. Q. Gao. Realization of Four-Pass I_2 Absorption Cell in 532-nm Optical Frequency Standard. *IEEE Trans. Instrum. Meas.*, 56(2):673–676, 2007.

[17] C. Degenhardt, H. Stoehr, C. Lisdat, G. Wilpers, H. Schnatz, B. Lipphardt, T. Nazarova, P.-E. Pottie, U. Sterr, J. Helmcke, and F. Riehle. Calcium optical frequency standard with ultracold atoms: Approaching 10^{-15} relative uncertainty. *Phys. Rev. A*, 72(6):062111, 2005.